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The Initiation of Brittle Fracture by Wedge Penetration

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1. Crack starting stress and crack driving stress.

There is a fundamental difference between the fracture of a completely brittle material such as glass and the brittle (cleavage) fracture of normally ductile low carbon steels. According to the available experience, the former process is fully governed by the Griffith crack propagation condition[1][2] which, in the simple case of a large plate under a uniaxial tension stress σ, containing a relatively short edge crack of length c, has the form

$$\sigma \approx \sqrt{\frac{E\alpha}{c}} \qquad (1)$$

where E is Young's modulus and α the specific surface energy of the surface of fracture. The crack starts to propagate when the (mean) tensile stress in the plate reaches the value of the propagating ("driving") stress given by eq. (1). If, in the course of its propagation, the mean stress rises above the value given by the Griffith equation, the crack accelerates; in the opposite case, it decelerates. It can be shown[3] that eq. (1) represents a necessary and sufficient condition of crack propagation in a fully brittle material.

In the brittle fracture of low carbon steels, two additional factors are of fundamental importance. X-ray photographs show[4] that a thin layer underneath the surface of fracture is plastically distorted; the effective thickness of this layer seems to be of the order of 1/100 in., and the plastic work p per unit of its area is roughly 2.10^6 erg/cm^2 under normal conditions. For this reason, the work needed for extending the surface of the crack walls by unit area is no longer α but $\alpha + p$; since surface energy of the common metals is of the order of 10^3 erg/cm^2, α is negligible beside p, and the Griffith condition of crack propagation has to be replaced by the equation suggested by the writer(5)

$$\sigma \approx \sqrt{\frac{Ep}{c}} \tag{2}$$

The second prominent feature of cleavage fracture in ductile steels is that even the modified crack propagation condition (2) is no longer a sufficient, but merely a necessary condition of brittle fracture. This can be recognized in the following way. It can be shown(3) that, for a fully brittle material, the Griffith equation (1) expresses the condition that the applied tensile stress, multiplied by the stress concentration factor of the crack, reaches the value of the molecular cohesion ("theoretical strength") of the material. In other words, when the mean tensile stress in the plate reaches the value given by eq. (1), the local tensile stress at the tip of the crack attains the maximum value that can be withstood by the intermolecular forces, and then the crack <u>must</u> start to propagate: eq. (1), therefore, is a <u>sufficient</u> condition of fracture in a uniformly stressed plate. With a ductile material like steel, this is no longer so. The local tensile stress at the tip of the crack can never reach the value of the molecular cohesion: long before even one per cent of this value would arise, plastic yielding occurs and redistributes the stress in the surroundings of the crack. On the other hand, cleavage fracture also occurs at a tensile stress far below the molecular cohesion, because, besides the large crack or notch from which brittle fracture starts, the material contains a multitude of microscopic or submicroscopic cracks which reduce its tensile strength to the low value of the "brittle strength" or "cleavage strength" which can be measured directly at low temperatures on specimens containing no visible crack or notch. Whether plastic yielding or cleavage fracture takes place first at the tip of the crack or notch depends on the relative magnitudes of the critical stresses for these two processes. The ordinary tensile test at room temperature shows that the yield stress of low carbon steels in uniaxial tension is lower than their cleavage strength, because yielding but no brittle fracture is observed in the test. If, nevertheless, cleavage fracture may occur at the tip of a crack or a notch, this is due to two factors acting simultaneously or individually:

First, after a slight plastic deformation has taken place at the tip of the crack, plastic constraint develops: the deformed region, being surrounded with material under lower stress, requires a higher tensile stress to overcome both its own resistance to plastic deformation, and the constraining influence of its surroundings. In this way, a triaxial state of tension arises, and the highest principal tensile stress may rise up to about 3 times the value of the yield stress Y in uniaxial tension[(4)(2)]. Consequently, if the cleavage strength B, though higher than Y, is lower than about 3Y, cleavage fracture can follow the local plastic deformation as the triaxiality of tension due to plastic constraint develops.

Second, it is known that the yield stress of low carbon steels increases exceptionally rapidly with the rate of deformation[6] (as also with decreasing temperature[7]). The cleavage strength does not show this high velocity-dependence; consequently, at very high rates of deformation the yield stress may rise above the cleavage strength, and then brittle fracture occurs without any preceding development of plastic constraint. This is possible only if the temperature is not too high (perhaps not above the lower region of the Charpy transition range), so that the cleavage strength is only moderately higher than the uniaxial yield stress; in the upper regions of the transition range, the velocity effect must be complemented by some plastic constraint. Now any plastic deformation that might occur around the tip of a fast running crack would have to take place at a very high rate: consequently, a fast-running crack may propagate as a cleavage crack without the necessity of significant plastic deformation occurring at its tip to produce plastic constraint.

Experiments show[8] that, for a reason that is not entirely understood, considerable amounts of local plastic deformation are needed for producing a cleavage crack at the end of a sharp notch or crack under static loading, so that the energy consumption for cleavage induced by plastic constraint is high. Genuinely brittle (low-energy) cleavage fracture occurs only if the tensile stress at yielding is raised to the level of the cleavage strength entirely or mainly by the velocity effect of a fast running crack. Since the crack cannot reach a high velocity at moderate rates of loading unless the work of propagation is covered by elastic energy released during the propagation, a necessary condition of genuinely brittle fracture in low carbon steel at moderate rates of loading is eq. (2): not unless the tensile stress exceeds the value given by (2) can the released elastic energy both cover the work of crack formation (as represented by the quantity p), and provide the kinetic energy for accelerating the crack.

It should be remarked that cleavage fracture often starts in service under the prolonged action of a static load without any significant plastic deformation (apart from that in the thin layer at the surface of fracture). The interesting possibilities for explaining this phenomenon will not be discussed here because the primary subject of the present paper is brittle crack propagation and initiation in certain laboratory tests in which such quasi-brittle crack initiation under static load has never been observed.

It follows, then, that cleavage fracture in low carbon steels requires the fulfillment of two sets of conditions:

a. The cleavage strength B must be low enough to be reached by raising the uniaxial yield stress Y through plastic constraint and/or the velocity effect of the running crack. If q (approx. = 3) is the highest plastic constraint obtainable by a crack, and v the ratio between the yield stress at very high rates of deformation to that at static testing rates,

$$B < q \cdot v \cdot Y \qquad (3)$$

is the condition for the possibility of cleavage fracture at the given temperature if constraint and velocity effect act combined; and

$$B < v \cdot Y \qquad (4)$$

is the condition of genuinely brittle crack propagation in which the velocity effect alone can produce cleavage fracture, without the necessity of plastic deformations to cause constraint.

b. If the crack starts at moderate rates of loading, the stress must be high enough to produce plastic deformation in a region large enough for developing the necessary plastic constraint; let σ_s be the value of the critical crack-starting stress under given conditions.

If the crack runs at high speed, so that the velocity effect is a governing factor in cleavage fracture, the condition for the applied stress to be high enough to keep the crack moving at high velocity (i.e., for providing the work of propagation from the released elastic energy) is

$$\sigma_d \approx \sqrt{\frac{Ep}{c}} \qquad (2)$$

where σ_d is the crack driving (propagating) stress.

It is easy to see that, under normal conditions, the crack starting stress must be higher than the crack driving stress. If it is assumed that the cleavage strength does not depend much on the rate of loading, the tensile stress at the tip of the crack must be more or less the same both in starting and in propagating. The applied stresses σ_s and σ_d necessary for producing the critical local stress at the tip of the crack must, then, be inversely proportional to the effective stress concentration factors in the two cases.

At first sight, it may appear unjustified to speak of (elastic) stress concentration factors in cases where plastic deformation takes place and levels down the stress at the tip of the crack; however, Neuber[9] has shown how the elastic stress concentration can be calculated in such cases, provided that the diameter of the plastically deformed region remains small compared with the length of the crack. He demonstrated that, if plastic deformation takes place in a region of radius R around the tip of the crack, the effective elastic stress concentration factor, defined as the ratio of the stress in the plastic region to the applied mean stress, is approximately equal to the stress concentration factor of a crack of the same length but of tip radius R in a purely elastic body. Since the elastic stress concentration factor in the latter case is approximately[10]

$$k \cong 2\sqrt{\frac{c}{R}} \qquad (5)$$

the stress required for obtaining the value of the cleavage strength at the tip of the crack is approximately proportional to the square root of the radius of the plastic region around the tip. Now this radius must be about equal to the thickness of the plastically distorted surface layer as revealed by X-ray diffraction if the crack runs at high velocity; as mentioned at the beginning of this section, the effective thickness of this layer is about 1/100 in. If, however, a cleavage crack is being started, the plastic deformation at the tip of the initial notch or crack is so extensive under laboratory conditions that it can be seen with the naked eye at some distance. Consequently, the radius of the plastically distorted region is some

10 or 20 times greater when the crack is started than at the tip of the running crack; according to eq. (5), then, if no other factors of importance are present, the starting stress σ_s must be roughly 3 or 4 times higher than the propagating stress. This conclusion is in fair agreement with the observation that the starting stress is usually quite close to the yield point which is of the order of 40,000 psi, while the driving stress seems to be between 10,000 and 15,000 psi[11][12].

The essential difference between the starting stress and the driving stress was the main reason why the fracture tests on wide plates carried out during the last war in connection with brittle fractures in welded ships were so remarkably unrevealing. In these tests, only the starting stress could be measured, and this was always quite close to the yield stress of the plate. The experiments gave no hint about the cause of brittle fractures that must have occurred at considerably lower stress levels.

2. Measurement of the crack propagating stress.

The distinction between the starting stress and the driving stress can be recognized directly from the fact that the fracture of a Charpy or Izod specimen may be quite ductile although, at the same temperature, brittle cracks can propagate in the material once they have run into it. This shows that the combination of notch constraint and impact velocity used in the conventional notch impact tests is not sufficient for starting the propagation of a crack in all conditions under which a cleavage crack can propagate in the material.

To the engineer, the driving stress is more important than the starting stress. A cleavage crack can start by some accidental circumstance which may be difficult to avoid (e.g., an underwater explosion), or in an accidentally too brittle part of the structure. However, this cannot lead to catastrophic failure of the structure if its bulk consists of material that cannot propagate cleavage cracks at the service temperature. This raises the question: is it possible to devise tests which, unlike the conventional notch-impact tests, include means for initiating a cleavage crack in all conditions under which such cracks may propagate? Such tests could reveal those combinations of stress, crack length, and temperature, at which crack propagation is possible in a given material. The simplest way of sending a "pre-fabricated" crack into a plate under tension would be to provide one of its edges with a notched flap in which a cleavage crack would be initiated by tension [5]; the tensile stress in the flap could reach the yield point and so it could initiate a brittle crack even if the stress in the main body of the specimen would be quite low.

The first experiments in this field have been carried out by T. S. Robertson[11][13][14]. He provided the edge of the plate with a saw-cut and started the propagation of a cleavage crack from its tip by a combination of a wedging impact and lowered temperature. The wedging force was exerted in the way indicated by Fig. 1: a blow on the round eyelet caused plastic deformation which forced apart the saw-cut inside the eyelet. As indicated in the figure, the specimen had a transverse temperature gradient, the side provided with

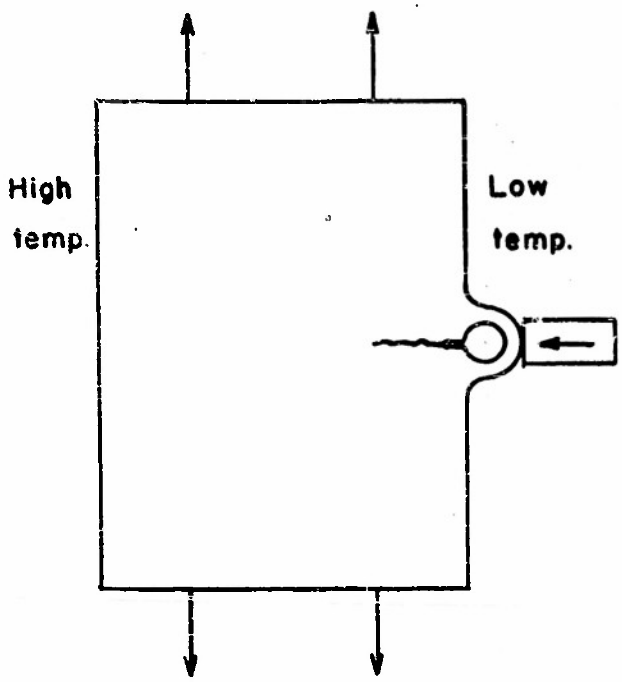

Fig. 1

the eyelet and the notch being colder than the opposite side; the cleavage crack initiated at the notch stopped in its way across the plate, and the temperature at the point of arrest could be regarded as the upper limit at which cleavage crack propagation was possible under the conditions of the experiment.

By repeating the experiment with different values of the tensile stress in the plate, Robertson could plot curves showing the dependence of the temperature limit upon the applied stress(14). Although some materials gave capricious curves, the majority behaved according to Fig. 2: There was usually a sharp temperature limit above which no stress could propagate a cleavage crack. Below this limit, the arrest temperature dropped very rapidly with decreasing stress; the relationship between the two quantities was often a straight line, as shown with full lines in Fig. 2. Occasionally, there was a step in the line, as indicated with dotted lines, and sometimes the type of the curve was quite different from those shown in Fig. 2. Robertson verified experimentally that the temperature limits of crack propagation were the same in plates of uniform temperature, so that the presence of a

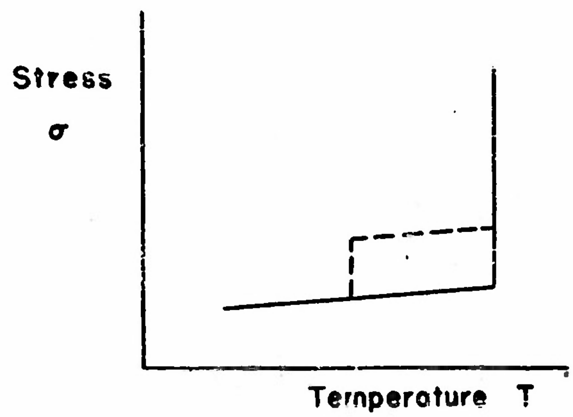

Fig. 2

temperature gradient did not introduce errors.

Recently similar experiments have been published by Feeley, Hrtko, Kleppe, and Northup[12]. In these, the notch was provided with a continuation consisting of a short cleavage crack before the experiment; after the plate was put under tensile stress, a wedge was driven into the notch by the impact of a bullet. It was found that the lower limit of the tensile stress below which no fracture occurred was remarkably independent of the length of the initial crack (including the notch) within a certain temperature interval inside the transition range; above and below this interval, the minimum fracture stress increased with increasing temperature. It should be added however, that the crack lengths varied only between 3/4 in. and 2-1/2 in. In addition, the energy of the wedge impact did not influence the minimum fracture stress if the kinetic energy of the bullet was above 100 ft-lb; below this value the fracture stress increased with decreasing impact energy. Finally, the minimum fracture stress did not depend on the temperature in the interval just mentioned within the transition range; outside this interval, it increased with the temperature.

3. The meaning of the Robertson curves.

As mentioned in connection with Fig. 2, the σ - T curves obtained by Robertson usually consisted of two different parts: a vertical, or nearly vertical one, and another which was slightly sloping against the horizontal. The probable interpretation of the vertical portion is that it represents the temperature limit above which the velocity effect (aided by the slight triaxiality of tension present around the tip of the running crack) is unable to raise the yield tension to the level of the cleavage strength. The meaning of the sloping part of the curve is less simple; the following interpretation seems to be fairly plausible.

Below the temperature limit given by the vertical part of the curve the propagation of the crack ought to be governed by a condition of the type of eq. (2). With increasing crack length c, the applied (mean) tensile stress required for propagation decreases if the plastic surface work p remains constant. In Robertson's experiments p was not constant: since the temperature increased in the direction of crack propagation, the value of p must also have increased as the crack ran into regions of increasing temperature. However, this effect could hardly overcompensate the rapid increase of c: almost certainly the stress required for further propagation must have dropped in the course of the propagation. If the crack nevertheless stopped before it arrived at the temperature limit of cleavage propagation, this could hardly have been due to anything but the sharp drop of load that accompanies the propagation of a crack if the testing machine cannot follow the elongation of the specimen. In experiments of Felbeck and the present writer it was often found that the crack stopped halfway through the plate (which was of uniform temperature) owing to load relaxation in the testing

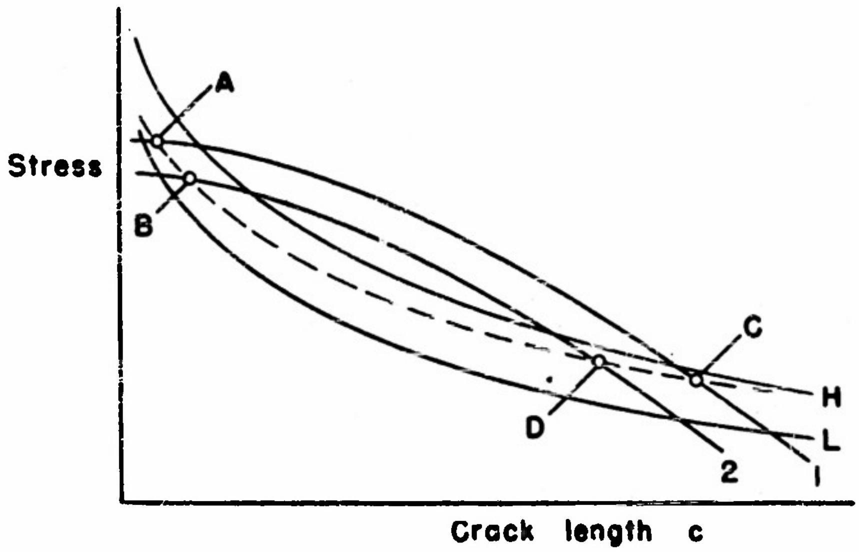

Fig. 3

machine[8]. What must have happened in Robertson's experiments is illustrated in Fig. 3. H is the curve representing the crack driving stress as a function of the crack length c, according to eq. (2); at a lower temperature the magnitude of p must be lower, and the corresponding curve L must lie below H. If L refers to the temperature at the cold side, and H to that at the hot side of the Robertson specimen, the actual curve of the driving stress will be given by the dashed transition curve between L and H. The drop of the mean tensile stress due to load relaxation is represented schematically by the curves 1 and 2, the former referring to a higher and the latter to a lower value of the initial stress. In drawing these curves it was assumed that the load drop overcompensates the decrease of the load carrying area during crack propagation;

this is to be expected for the hydraulic loading device used by Robertson[14].

Up to the point A of curve 1, or point B of curve 2, there can be no crack propagation unless the applied stress is complemented by the wedging impact upon the eyelet. From A to C, and from B to D, the crack propagates with acceleration since the applied mean stress exceeds that demanded by the crack propagation condition; at C or D, the crack decelerates and later stops. With a higher initial stress (curve 1) the final length of the crack is obviously greater than with a lower stress (curve 2); since, however, longer cracks end at points of higher temperature in the Robertson experiment, the temperatures of arrest in the region below the critical temperature limit must increase with the initial stress. If this interpretation is substantially correct, the gently sloping part of the Robertson curves may reflect the behavior of the testing equipment rather than significant properties of brittle fracture.

4. The experiments of Feeley, Hrtko, Kleppe, and Northup.

One of the remarkable features of these experiments is the observed independence of the minimum fracture stress from the initial crack length, the impact energy, and the temperature, in considerable ranges of these variables. It is an important question whether this independence is a fundamental property of brittle fracture or a more or less accidental consequence of the experimental conditions: in the first case, the observed minimum fracture stress (10,000 psi) could be the basis of a design stress the use of which would safeguard the structure against the possibility of brittle fracture.

However attractive this may seem to the designing engineer, a simple consideration shows that it could hardly be reconciled with elementary facts of solid mechanics. A crack propagates when local fracture occurs at its tip: this cannot depend on anything but the local stresses and strains. These, however, are by no means determined by the mean tensile stress in the plate alone: the length of the crack is equally important. It determines the elastic stress concentration factor by which the applied stress has to be multiplied in order to obtain the local stress at the crack. If the stress is so high that the entire plate is yielding, there is, of course, no elastic stress concentration and the stress at the crack tip is determined by the plastic constraint factor which does not depend much on the length of the crack. However, in the experiments considered the tensile stress was between 1/4 and 1/3 of the yield stress, so that the entire plate must have been purely elastic with the exception of a small plastic region at the tip of the crack. Under such circumstances, Neuber's theorem, as outlined in Section 1, can be applied; in connection with the elastic stress concentration factor eq. (3), it leads to the conclusion that the applied stress at which the amount of plastic yielding needed for developing the plastic constraint takes place must be approximately inversely proportional to the length of the initial crack.

This argument seems so inevitable that it probably rules out the possibility of a crack propagating stress being fundamentally independent of the length of the crack. The question is, then: What is the cause of the independence of the minimum fracture stress as observed by Feeley, Hrtko, Kleppe, and Northup, from the crack length, the impact energy, and the temperature in certain ranges of these variables?

To begin with, it is by no means certain or even probable that the minimum fracture stress observed by Feeley and his associates is a crack propagating stress. In Robertson's experiments there is no doubt that a moving crack has been created by the impact upon the cooled notched eyelet, and that the failure of the experiments to give information about the magnitude of the driving stress is due merely to the fact that the applied load cannot be measured reliably when the crack starts to run. In the experiments of Feeley and his associates, on the other hand, no crack propagation can be observed unless complete fracture occurs. This suggests strongly the possibility that, for some reason, the wedge-impact method of starting the crack propagation may not be as effective as the eyelet-impact method, so that the stress that has to be applied for starting the crack under wedge impact is higher than the stress required for propagating it once it has been started. In other words, the fact that the crack does not move unless it runs through the plate suggests that the minimum fracture stress observed is a crack starting stress, not a driving stress. A detailed analysis given in Section 6 supports this possibility. Before dealing with it, however, a relatively trivial possibility of explaining the independence of the driving stress from the crack length in a limited range of crack lengths should be discussed.

5. Influence of the finite specimen size.

If the plate is very wide compared with the length of the crack, the elastic stress concentration factor is given approximately by eq. (3): it is proportional to the square root of the length of the crack. Fig. 4 shows the opposite limiting case of a plate containing two symmetrical edge cracks so deep that the remaining width of the plate between their tips is small compared with the radius of curvature of the tips. In this case, the stress concentration is small if (as usual) the nominal stress is referred to the remaining cross section between the cracks, and it converges to 1 as the distance between the crack tips becomes vanishingly small compared with the tip radius which is considered to be constant (this corresponds to the fact that the tip radius of an atomically sharp crack is of the order of the interatomic spacings and does not change with the length of the crack). Figure 5, after Neuber[9], shows the curve 1 representing the dependence of the stress concentration factor of a relatively shallow crack upon the crack length, and the curve 2 giving the same dependence for a deep crack (i.e., a crack the length of which approaches half of the width of the plate and which faces another crack situated symmetrically at the opposite edge of the plate). The complete dependence of the stress concentration factor upon the depth of the crack must be given by a curve (fully drawn) which converges at the two limiting points towards the asymptotic curves; it must have a maximum at an intermediate point,

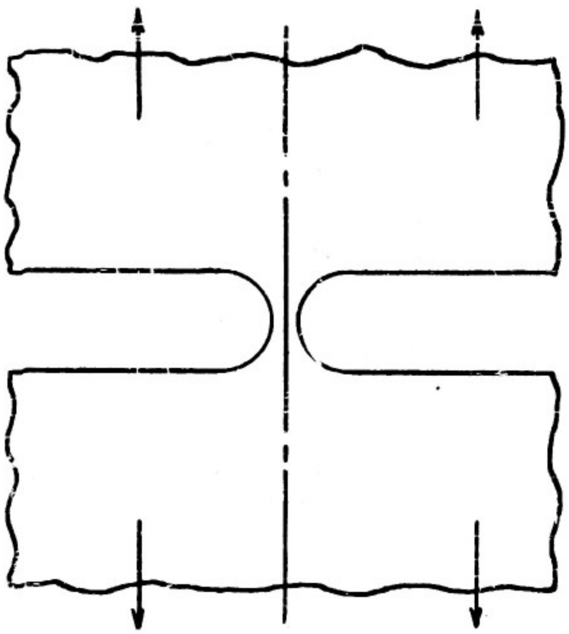

Fig. 4

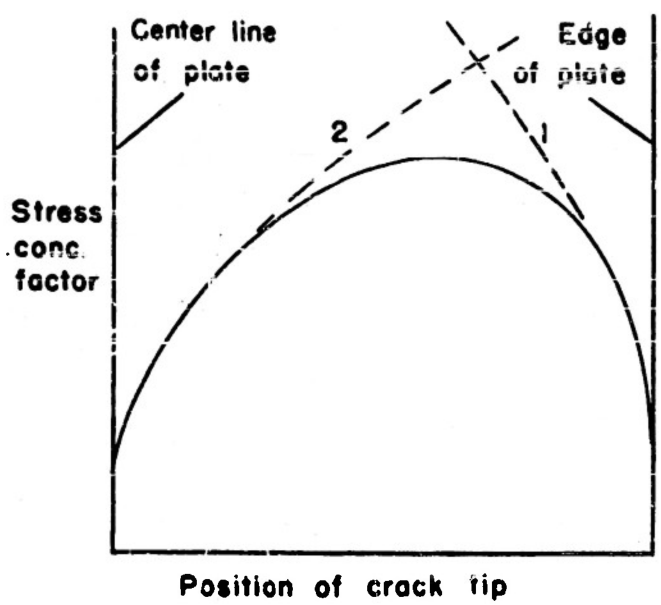
Center line
of plate
Edge
of plate
2
1
Stress
conc.
factor
Position of crack tip

Fig. 5

and the dependence of the stress concentration factor upon the crack length must be small in the region around the maximum. According to Neuber's calculations, the maximum occurs in a symmetrically notched plate as used in the experiments of Feeley, Hrtko, Kleppe, and Northup when the depth of the crack is about 1/6 of the full width of the plate. In the experiments, the crack length varied between 3/4 in. and 2 in., and the plate widths used were 6 in., 10 in., and 16 in.; thus, the ratio of the crack length to the plate width varied between about 1/20 and 1/3. The cases investigated must have been, therefore, just around the top of the fully drawn curve in Fig. 5 where the influence of the crack length upon the driving stress is very small. It seems nevertheless that the effect of the crack length ought to have been noticeable if the finite width of the plate had been the only cause of the insensitivity of the fracture stress to the crack length.

The effect of the finite width has been discussed in greater detail on a somewhat different basis by Professor M. Gensamer[15]; his final conclusions agree, on the whole, with the present ones.

Since the effect of the finite width is unlikely to explain fully the observations, in what follows a simple general analysis of the condition of brittle crack propagation under the combined influence of a tensile stress and a wedge pressed into a crack will be given.

5. Crack propagation under tension combined with wedge penetration.

As a preparation for the discussion of the experiments of Feeley and his associates, the Griffith theory of crack propagation in a fully brittle plate should be extended to the case of a plate being under tension while at the same time a wedge is being forced into

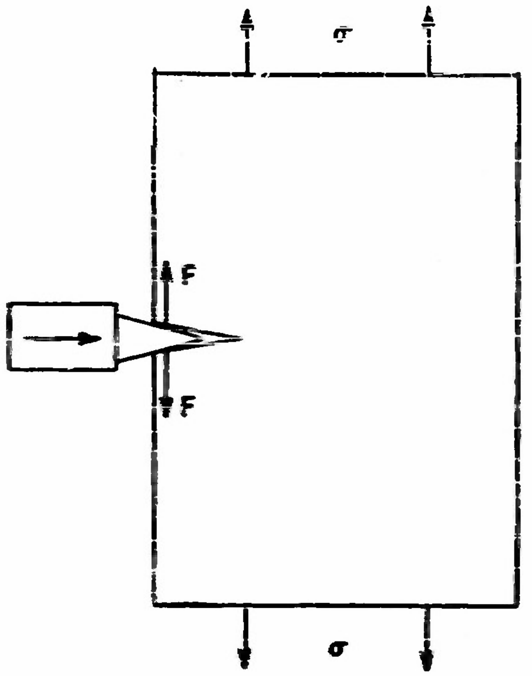

Fig. 6

a sharp edge crack of length c (Fig. 6).

The force exerted by the wedge upon the walls of the crack has both vertical and horizontal components. However, the horizontal (transverse) ones are of minor influence; in what follows, they will be disregarded. Let F be the vertical wedge force per unit thickness of the plate; according to the experimental conditions, it may be concentrated in small areas of the crack walls adjacent to the edge of the plate, or distributed in some manner over the walls. In the experiments of Feeley and associates, the wedge force was distributed over areas that must have been roughly 1/8 to 1/4 of the area of a crack wall; this was a consequence of the plastic squeezing of the crack walls by the wedge. For the following consideration, it will be assumed that the vertical wedge force is distributed uniformly over the crack walls; if it is not, a correction factor f of order unity can be applied so that the effects of the wedge force F are equivalent to those of a uniformly distributed force f . F. Such a uniformly distributed force amounts to a uniform pressure fF/c acting upon the crack walls.

This pressure can be removed by immersing the entire plate into a liquid under the hydrostatic tension fF/c. The superposition of a hydrostatic tension would make no difference if the plastic behavior of the plate were to be investigated; in the present case, however, it produces an additional tensile stress fF/c in every surface element at every point of the plate. This can be deducted ultimately; in addition, it is easily seen that its effect is small if the stress concentration factor of the crack is large compared with 1. With the hydrostatic tension superposed, all sides of the plate are exposed to an additional tensile stress fF/c. The stresses acting on the faces and the side edges of the plate cannot have much influence on the propagation of the crack; consequently, the main effect of the superposed tension (in addition to removing the wedge force from the crack walls) is to create an additional tensile stress fF/c perpendicular to the crack, superposed to the tensile stress applied to the plate in addition to the wedge force. The added stress fF/c increases the tensile stress at the tip of the crack by q . fF/c, where q is the stress concentration factor of the crack. If it is considerably larger than unity, the superposed hydrostatic tension fF/c is small compared with the additional tensile stress it creates at the tip of the crack, so that the final deduction of fF/c can be omitted. This will be done in the present case. Since, for the steel plates in the experiments discussed, fracture occurred under tensile stresses as low as 1/4 of the yield stress and probably 10 or 15 times less than the cleavage strength, the effective stress concentration factor of the crack must be of the order 10 or 15, and so the omission of the superposed hydrostatic pressure causes an error less than 10 per cent.

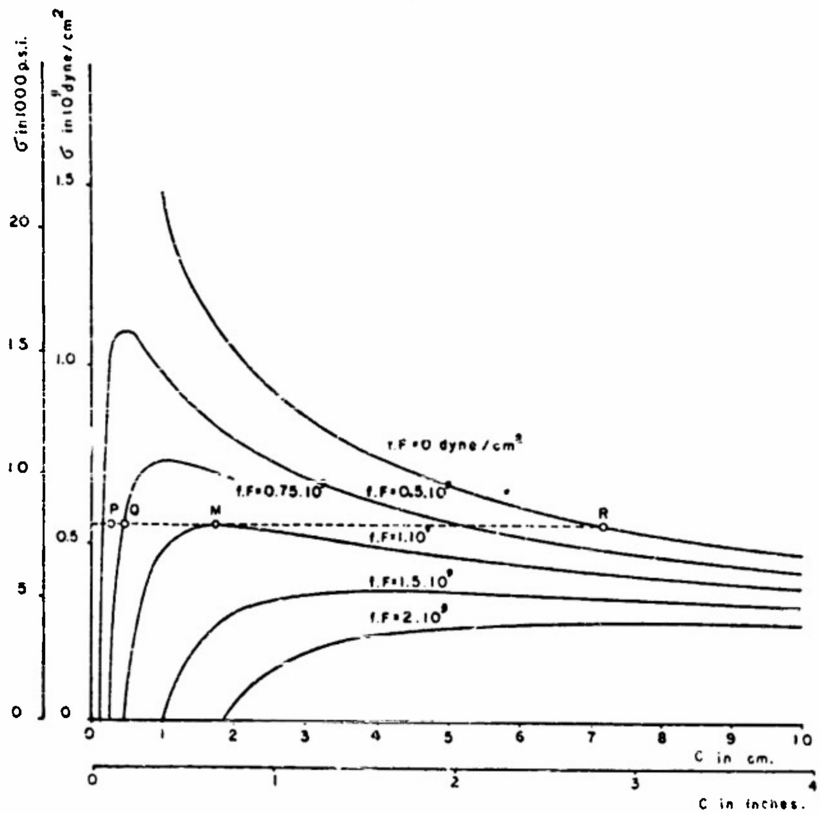

Fig. 7

The result of this consideration is that the wedge force F is approximately equivalent to an additional tensile stress fF/c. Without it, the crack propagation condition would have the Griffith form eq. (1); if the wedge force is applied, the condition becomes

$$\sigma + fF/c \approx \sqrt{\frac{E\alpha}{c}}$$

or

$$\sigma \approx \sqrt{\frac{E\alpha}{c}} - \frac{fF}{c}. \qquad (6)$$

Fig. 7 shows σ plotted as a function of the crack length c for several values of the wedge force F. With F = 0, the square-root hyperbola representing the Griffith eq. (1) results: with increasing values of the wedge force, the curves F = const. move downwards. All curves except that for F = 0 have a maximum at a value of c determined by

$$d\sigma/dc = -\tfrac{1}{2}\frac{(E\alpha)^{1/2}}{c^{3/2}} + \frac{fF}{c^2} = 0 ;$$

hence,

$$\sqrt{c_m} = \frac{2fF}{(E\alpha)^{\frac{1}{2}}} \qquad (7)$$

If this is introduced into (6), the value of σ at the maximum is obtained as

$$\sigma_m = \frac{E\alpha}{2fF} - \frac{E\alpha}{4fF} = \frac{E\alpha}{4fF} \qquad (8)$$

The curves F = const. intersect the abscissa axis before rising to the maximum. The point of intersection represents the length to which a crack can be driven be a wedge force alone, without an applied tension; eq. (6) and Fig. 7, therefore, include a crude theory of knife penetration into a large solid block. According to eq. (7), a finite wedge force can cause finite penetration only, if the block is infinitely large. It is remarkable that the crack length produced by a wedge increases with the wedge force. The longer the crack, therefore, the greater the wedge force necessary to propagate it further; infinitesimally short cracks can be extended with infinitesimally small wedge forces. This behavior, however natural, is contrary to the mental habit acquired in the Griffith theory where large cracks were easier to propagate.

Suppose now that a constant tensile stress is applied to the plate and a wedge is gradually pressed into the crack; the wedge force should be capable of increasing to a certain maximum amount but not beyond this. Let the applied stress correspond to the level of the dashed line in Fig. 7, and let it be assumed that the initial length of the crack is the abscissa of the point P. If the maximum possible wedge force is such that $fF = 0.75 \cdot 10^9$ dyne/cm^2 (see Fig. 7), it can extend the crack to the point Q but not more. If, however, the wedge force can be increased to 1.10^9 dyne/cm^2, the point M at the maximum of the curve $fF = 10^9$ dyne/cm^2 can be reached. Any further extension of the crack would take place at a decreasing wedge force; consequently, the maximum of a curve F = const. is a point of instability at which crack propagation under falling wedge force and thus fracture can take place. It does not take place under all circumstances, but only if the force applied to the wedge is always maintained at, or above, the value corresponding to the F = const. curve that goes through the point of which the ordinate is the applied tensile stress and the abscissa the current value of the crack length. When this representative point arrives at the Griffith hyperbola (point R), the wedge force may disappear, and the applied stress alone is capable of continuing the propagation of the crack.

A remarkable point is that the length of the initial crack does not matter at all if only it is shorter than the abscissa at the maximum of the F = const. curve of which the horizontal representing the applied stress is a tangent. Under this condition, the minimum fracture stress for a given wedge force is quite independent of the length of the initial crack.

So far, only crack propagation in a fully brittle material has been considered. Can the results be applied to normally ductile but notch-brittle steels after replacing the surface energy α by the plastic surface work p? In this way, eq. (5) would become

$$\sigma \approx \sqrt{\frac{Ep}{c}} - \frac{fF}{c} \qquad (5a)$$

and eqs. (7) and (8) would change into

$$\sqrt{c_m} = \frac{2fF}{(Ep)^{\frac{1}{2}}} \qquad (7a)$$

and

$$\sigma_m = \frac{Ep}{4fF} \qquad (8a)$$

The preceding considerations have led to the view that low carbon steel has two alternative crack propagation conditions. The first applies to a crack at rest; it gives the "starting stress" required for setting it in motion. The second applies to a rapidly extending crack; it gives the "driving stress" necessary for maintaining the velocity of propagation. The driving stress is the applied stress needed for producing at the tip of the crack a small plastically deformed region in which the local stress rises to the value of the cleavage strength by the combined action of the high rate of deformation and of a (relatively small) triaxiality of tension. The crack starting stress, on the other hand, has to produce a higher plastic constraint because, in the absence of a high strain rate, the triaxiality of tension alone has to raise the stress to the cleavage level; for this purpose, it has to induce plastic deformation in a region of greater radius. In view of Neuber's theorem (Section 1), therefore, the two propagation conditions differ mainly in that the starting condition demands the creation of a larger region of plastic deformation. This requires a higher value of the applied stress because, according to Neuber, the tip radius of the equivalent crack in the purely elastic case (see Section 1) is then greater and so the stress concentration factor for a crack of given length lower, whereas the stress that must be reached at the tip of the crack is equal to the cleavage strength both for the resting and for the moving crack.

It can be shown that the Griffith type eqs. (1) and (2) are equivalent to conditions demanding the attainment of a critical fracture stress at the tip of the crack[3]; instead of using atomic quantities such as the tip radius of a cleavage crack and the molecular cohesion, they express the same propagation condition in terms of macroscopic quantities (Young's modulus E and the specific work of crack wall formation α or p). That the starting of a crack requires more extensive plastic deformations than its further propagation will be reflected in a higher value for p in eqs. (6a), (7a), and (8a) when these equations are used for obtaining the starting condition; instead of the value of 2.10^9 erg/cm^2, valid for the running crack, the magnitude of p may be 10 or 50 times higher when the crack is being started. This means that the Griffith hyperbola in Fig. 7 will be raised by a factor of, say, 3 to 7, and the curves $\sigma = \sigma(c)$ for F = const. will rise by the same amount as the corresponding points of the Griffith curve because the difference between the ordinates of the latter and those of the former (see eq. (6)) is $f\Upsilon/c$, i.e., independent of p. Eqs. (7a) and (8a) show that the abscissa of the maximum of the $\sigma - c$ curve will be smaller, and the value σ_m of the maximum larger. This means, according to Fig. 7, that the crack requires for starting higher values of the wedge force, or of the tensile stress, or both, than for its further propagation after it has gathered speed. In other words, even with the use of a wedge a starting difficulty has to be overcome, and the combinations of tensile stress and wedge force needed for starting the crack are as a rule much more powerful than is necessary for its subsequent propagation.

This raises the question whether the use of a wedge impact can really fulfill its intended purpose of sending a running crack into the plate which would propagate or stop according to whether the applied tensile stress is above or below the value of the driving stress. The hope that this would be possible was based on the simple picture that the wedge impact would start the crack, give it a sufficiently high velocity, and then disappear before the initial crack length would increase significantly. Fig. 7, however, shows that this is far from being true. The wedge force must act until the crack has reached the critical length at which the applied stress alone can propagate it (point R in Fig. 7). This critical length, however, is in general much greater than the initial length of the crack: with the value $p = 2.10^6$ erg/cm^2 valid for the running crack and the tensile stress of 10,000 psi $\cong 6\cdot9 \, . \, 10^8$ dyne/cm^2, eq. (2) gives

$$c \cong \frac{Ep}{\sigma^2} = \frac{2\cdot2 \, . \, 10^{12} \, . \, 2\cdot10^6}{4\cdot75 \, . \, 10^{17}} \cong 9\cdot3 \text{ cm} \approx 3.6 \text{ in.} \qquad (9)$$

If the initial crack is shorter, it has to be extended to this length by the combined action of the wedge force and the applied tension before the latter can take over alone. If the wedge impact is not powerful enough, the applied tension must be increased in order to take over the propagation of the shorter crack that the wedge is capable of producing. If the impact is stronger, however, it does not follow necessarily that it can create a longer crack which then can be driven by a lower stress. An important factor is the duration of the impact. If the wedge force disappears between M and R in Fig. 7, the crack length produced is less than at R, and the crack needs for its propagation a higher stress.

The conditions of the wedge impact experiment are, then, far more complex than was anticipated when such experiments were planned. First, the wedge force may change the length of the initial crack drastically before it disappears and leaves it to the plate stress to propagate the crack. Unfortunately the driving stress cannot be obtained unless the length of the crack at the moment of the disappearance of the wedge force is known, and this can at present only be estimated, as in eq. (9), from a value of p obtained elsewhere. Second, at the moment when the wedge force vanishes the crack has a kinetic energy which may extend it further, so that there may be a substantial increase of the crack length before the applied stress takes over the propagation. Third, the load of the testing machine drops by relaxation as the crack opens up before it reaches the take-over length: at the moment when the applied stress takes over the propagation, therefore, both the length of the crack and the current value of the mean tensile stress are unknown: both may be very different from the initial values of c and σ. Unless a radical improvement of the experimental method can be

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